Binary Maths

Introduction

Now that we have some way of representing small binary values (from 0 to 15) in a nibble, we can start to explore how we might add the values of two separate nibbles.

An easy start:

Let's say we have two nibbles, nibble x and nibble y. We want to add their values together, and put the answer into nibble z.

Nibble x contains the value 1000 [or 8 in decimal]
Nibble y contains the value 0010 [or 2 in decimal]

So (this is a bit like going to binary primary school, but bear with me):

Nibble x	1	0	0	0
Nibble y	0	0	1	0
Nibble z	**1**	**0**	**1**	**0**

Pretty easy: 0+0=0 and 0+1=1 the whole way through – and we can back-check – that leftmost 1 is in the 8 column; there's a zero in the four column, a 1 in the 2 column, and no unit. 2+8=ten.

Remember

8s	4s	2s	1s

Let's carry on:

Simple so far, but what if we want to add nibble x and nibble y, where:

Nibble x contains the value 1010 [or ten in decimal]
Nibble y contains the value 0010 [or 2 in decimal]

Now, just like in decimal, I have to **carry** a value to the next column (remember we can only use 1 and 0 as our values).

Step 1

Nibble x	1	0	1	0
Nibble y	0	0	1	0
Nibble z				**0**
"carry" value				

All nice and easy

Step 2

Nibble x	1	0	1	0
Nibble y	0	0	1	0
Nibble z				**0**
"carry" value				

We cannot put 2 in here …

Nibble x	1	0	1	0
Nibble y	0	0	1	0
Nibble z			**0**	**0**
"carry" value		1		

So we put a **zero in here …**

… and a **two to carry** as a **one** in the next column

Step 3

Nibble x	1	0	1	0
Nibble y	0	0	1	0
Nibble z		**1**	**0**	**0**
"carry" value		1		

0 plus 0 is 0, but then we have 1 carried-over from the previous column …

Step 4

Nibble x	1	0	1	0
Nibble y	0	0	1	0
Nibble z	**1**	**1**	**0**	**0**
"carry" value		1		

Simple enough, 1+0=1.

So, an eight and a four – that's twelve. And yes, that's the right answer.

Carrying it a bit further:

Let's try to add 3+3

3 as a nibble is 0011

Step 1

Nibble x	0	0	1	1
Nibble y	0	0	1	1
Nibble z				**0**
"carry" value			1	

All OK so far – 1 + 1 is 2, so enter 0 and carry the 2 as a 1 in the next column.

Step 2

Nibble x	0	0	1	1
Nibble y	0	0	1	1
Nibble z			**1**	**0**
"carry" value		1	1	

1+1 is 2, **but** we also have a carry digit, making 3, so we enter 1, and carry the 2 as a 1 in the next column

Step 3

Nibble x	0	0	1	1
Nibble y	0	0	1	1
Nibble z	**0**	**1**	**1**	**0**
"carry" value		1	1	

0+0=0, add our carried digit from the previous column, and we get 1 …
In the leftmost column 0+0=0

Now, in our binary answer we have a 1 in the 4 column, and a 1 in the 2 column; 4+2=6 – and in decimal 3+3=6

EXERCISES

Here are some simple sums for you to try.
First convert the decimal values into binary; then insert them into the grid and add them up; then convert them back to decimal to check…

Remember

8s	4s	2s	1s

Add 3 to 4

Nibble x				
Nibble y				
Nibble z				

Add 5 to 8

Nibble x				
Nibble y				
Nibble z				

Add 3 to 6

Nibble x				
Nibble y				
Nibble z				
“carry” value				

Add 3 t0 12

Nibble x				
Nibble y				
Nibble z				
“carry” value				

You should be on a roll by now…

OK

Now let's try:

Add 7 to 13

Nibble x				
Nibble y				
Nibble z				
"carry" value				

Describe what happened:

Explain (or suggest) how you might get around this: